## QUESTION 1

Solve the equation

$$\frac{\sin\theta}{\cos\theta+1}+\frac{\cos\theta}{\sin\theta+1}=1$$

for $0^\circ \le \theta \le 360^\circ$.

$\Rightarrow$ (*) $\cos\theta \neq -1$, $\sin\theta \neq -1$

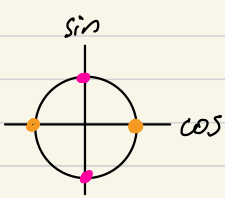
$$\frac{\sin\theta}{\cos\theta+1}+\frac{\cos\theta}{\sin\theta+1}=1;$$

$$\frac{\sin\theta(\sin\theta+1)+\cos\theta(\cos\theta+1)}{(\sin\theta+1)(\cos\theta+1)}=1; \quad (*)$$

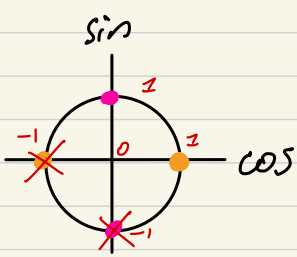
$$\sin^2\theta+\sin\theta+\cos^2\theta+\cos\theta=\sin\theta\cos\theta+\sin\theta+\cos\theta+1$$

$$\underbrace{\left[\sin^2\theta+\cos^2\theta\right]}_{\parallel\ 1}=\sin\theta\cos\theta+1;$$

$$\Rightarrow \sin\theta\cos\theta=0 \Rightarrow \begin{cases}\sin\theta=0\\ \cos\theta=0\end{cases}$$




But




Hence $\theta = 0^\circ, 90^\circ$

**QUESTION 2** Solve $17 = 4\cos^2\theta + 17\sin\theta$ where $0° < \theta < 360°$.

$4\cos^2\theta + 17\sin\theta = 17;$

$4(1-\sin^2\theta) + 17\sin\theta - 17 = 0;$

$-4\sin^2\theta + 4 + 17\sin\theta - 17 = 0;$

$-4\sin^2\theta + 17\sin\theta - 13 = 0; \quad | \times -1$

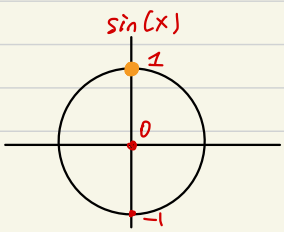
$4\sin^2\theta - 17\sin\theta + 13 = 0;$

⊙ let $q = \sin\theta$, $\boxed{|q| < 1}$ hence

$4q^2 - 17q + 13 = 0; \quad (4q-13)(q-1) = 0$

$\Rightarrow \begin{cases} q = \frac{13}{4} & \text{reject } |\sin\theta| < 1 \\ q = 1 & \Rightarrow \end{cases}$

$\Rightarrow \sin\theta = 1$ hence $\theta = 90°$

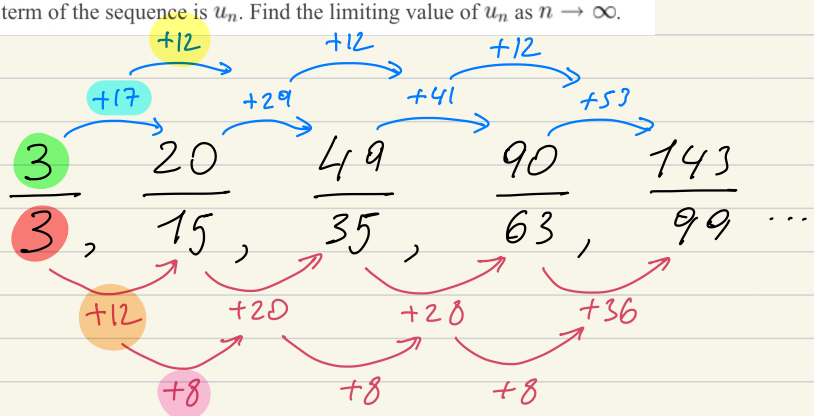


## QUESTION 3

Here are the first five terms of a sequence:

$$\frac{3}{3}, \frac{20}{15}, \frac{49}{35}, \frac{90}{63}, \frac{143}{99}$$

The $n^{\text{th}}$ term of the sequence is $u_n$. Find the limiting value of $u_n$ as $n \to \infty$.




TOP: $2a = 12 \Rightarrow a = 6$

$3a + b = 17 \Rightarrow b = 17 - 3a = 17 - 3\times 6 = -1$

$a + b + c = 3 \Rightarrow c = 3 - a - b = 3 - 6 + 1 = -2$

$\Rightarrow$ top $= 6n^2 - n - 2$

BOTOM: $2a = 8 \Rightarrow a = 4$

$3a + b = 12 \Rightarrow b = 12 - 3a = 12 - 3\times 4 = 0$

$\Rightarrow b = 0$

$a + b + c = 3 \Rightarrow c = 3 - a - b = 3 - 4 - 0 = \underline{-1}$

$\Rightarrow$ Bottom $= 4n^2 - 1$

$$\lim_{n\to+\infty} \frac{6n^2-n-2}{4n^2-1} \overset{:n^2}{=} \lim_{n\to+\infty} \frac{6-\frac{1}{n}-\frac{2}{n^2}}{4-\frac{1}{n^2}} =$$

($\frac{1}{n}\to 0$, $\frac{2}{n^2}\to 0$, $\frac{1}{n^2}\to 0$)

$$= \frac{6}{4} = \frac{3}{2};$$

**QUESTION 4** Given that $f(x) = x^3 - kx^2 - 13x + 5k$ and $f(5) = 0$, find the other two solutions to $f(x) = 0$.

$\Rightarrow f(5)=0;\quad 5^3 - k\times 5^2 - 13\times 5 + 5\times k = 0;$

$125 - 25k - 65 + 5k = 0;\quad -20k + 60 = 0;$

$k = 3$

If $f(5)=0$ then $(x-5)$ is a factor

$f(x) = x^3 - 3x^2 - 13x + 15$

$x^3 - 3x^2 - 13x + 15 \equiv (x-5)(Ax^2 + Bx + C)$

$\equiv Ax^3 + Bx^2 + Cx - 5Ax^2 - 5Bx - 5C$

$\equiv Ax^3 + (B-5A)x^2 + (C-5B)x - 5C$

$\Rightarrow A = 1;\quad -5C = 15 \Rightarrow C = -3$

$B - 5A = -3;\quad B = 5A - 3 = 5 - 3 = 2 \Rightarrow B = 2$

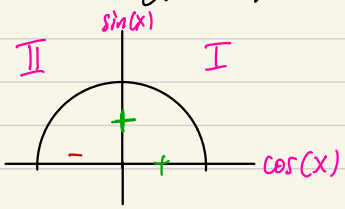
$$f(x) = (x-5)(x^2+2x-3) = (x-5)(x+3)(x-1)$$

$$\Rightarrow \quad x=5 \quad \text{OR} \quad x=-3 \quad \text{OR} \quad x=1$$

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$\sin x = \frac{\sqrt{11}}{6}$ and it's **obtuse** → hence $x° > 90°$

What is $\cos x = ?$




## QUESTION 5

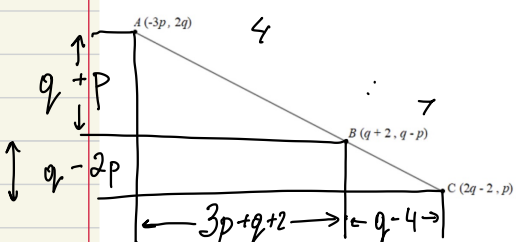
In II quadrant, $\cos(x) < 0$

Hence $\cos(x) = -\sqrt{1-\left(\frac{\sqrt{11}}{6}\right)^2} = \frac{-5}{6}$

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## QUESTION 6

*ABC* is a straight line where *BC* is 20% of *AC*. In the diagram below, the coordinates of *A*, *B* and *C* are shown. *p* and *q* are positive numbers.




Find the value of *p* and *q*.

4 : 1

Hence

$$\frac{3p+q+2}{q-4} = \frac{4}{1}$$

$$3p+q+2 = 4q-16$$

$$3p-3q = -18;$$

$$BC = 20\%(AC) = \frac{1}{5}AC;$$

$$\frac{BC}{AC} = \frac{1}{5} \Rightarrow \frac{AB}{AC} = \frac{4}{5}$$

$$\Rightarrow AB:BC = 4:1$$

$$\frac{q+p}{q-2p} = \frac{4}{1}; \qquad \vec{q}+\vec{p} = 4q - 8p$$

$$\boxed{3q - 9p = 0}$$

$$\begin{cases} 3p - 3q = 0 \\ -9p + 3q = -18 \end{cases} \oplus \qquad -6p = -18 \Rightarrow p = 3$$

$$9p = 3q; \qquad q = 3p = 3 \times 3 = 9 \Rightarrow q = 9$$

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The transformation matrix $\mathbf{P}^3$ represents a clockwise rotation of $150^o$ about the origin.

$\mathbf{Q} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

The unit vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is transformed by the matrix $\mathbf{QP}$ to give the vector $\begin{pmatrix} -\cos\theta \\ \cos\alpha \end{pmatrix}$.

Find the value of $\theta$ and $\alpha$.

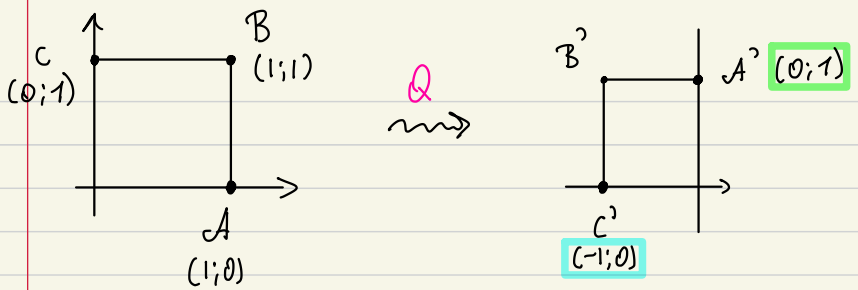
## QUESTION 7

$P^3$ means $P$ is done 3 times in a row

hence $P$ is a rotation $50°$ clockwise around the origin.

$\frac{150°}{3}$

Now, what is $Q$?

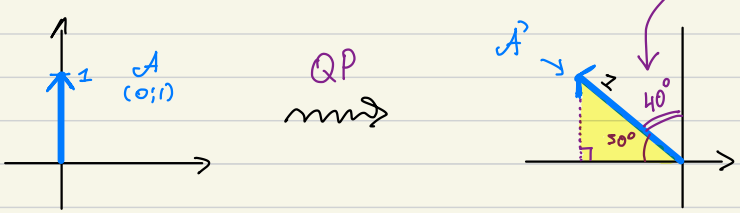


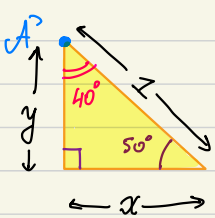

hence Q is a rotation 90° anticlockwise around origin

QP means P is done first then Q

hence 50° clockwise + 90° anticlockwise = 40° anticlockwise

(P) (Q) (QP)




$\cos(50°) = \frac{x}{1} \Rightarrow x = \cos(50°)$

$\cos(40°) = \frac{y}{1} \Rightarrow y = \cos(40°)$

$\Rightarrow A' \left(-\cos(50°) ; \cos(40°)\right)$

to give the vector $\begin{pmatrix} -\cos\theta \\ \cos\alpha \end{pmatrix}$. $\Rightarrow$ $\theta = 50^\circ$, $\alpha = 40^\circ$

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The nth term of a sequence is n^2 + n

two consecutive terms in the sequence have a difference of 32

work out the two terms

## QUESTION 8

Hence $\left[(n+1)^2 + (n+1)\right] - \left[n^2 + n\right] = 32$

$n^2 + 2n + 1 + n + 1 - n^2 - n - 32 = 0;$

$2n + 2 - 32 = 0; \quad 2n = 30; \quad n = 15$

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Simplify:

## QUESTION 9

$$\frac{(x^2 - xy)^2 - (xy - y^2)^2}{(x+y)}$$

(*) $a^2 - b^2 = (a-b)(a+b)$

$$\frac{(x^2-xy)^2 - (xy-y^2)^2}{(x+y)} = \frac{\left[(x^2-xy) - (xy-y^2)\right]\left[(x^2-xy) + (xy-y^2)\right]}{(x+y)} =$$

$$= \frac{(x^2 - 2xy + y^2)(x^2 - y^2)}{(x+y)} = \frac{(x-y)^2 (x-y)(x+y)}{(x+y)}$$

$$= (x-y)^3$$

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A function $f$ is defined by $f(x) = x^2 - 3x - 4$ and $-2 \leq x \leq 6$.

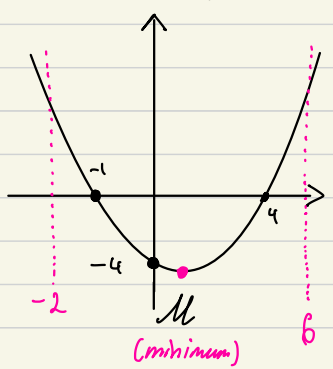
Find the range of $f(x)$.

**QUESTION 10**

$$f(x) = x^2 - 3x - 4 = (x-4)(x+1)$$

$+4; -1$

So $f(x)$ goes through $(0; -4)$; $(4; 0)$; $(-1; 0)$




$$M_x = \frac{-b}{2a} = \frac{-(-3)}{2 \times 1} = \frac{3}{2};$$

$My = f(Mx) = \left(\frac{3}{2} - 4\right)\left(\frac{3}{2} + 1\right) = \frac{-25}{4}$ ;

Hence the smallest value of is $My = \frac{-25}{4}$

The maximum value can be either in $f(-2)$ or $f(6)$

$f(-2) = [-2-4][-2+1] = 6$

$f(6) = (6-4)(6+1) = \underline{\underline{14}}$ bigger

Hence $\frac{-25}{4} \leq f(x) \leq 14$

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1.Prove $\tan^2\theta + \cos^2\theta = \tan^2\theta \sin^2\theta + 1$

2. Let $f(x) = \sqrt{\frac{-1}{x}}$

a. Find a suitable domain for $f$

b. Find the range of $f$

**QUESTION 11 & 12**

① RHS: $\tan^2\theta \sin^2\theta + 1 = \frac{\sin^2\theta \times \sin^2\theta}{\cos^2\theta} + 1$

$= \frac{\sin^4\theta + \cos^2\theta}{\cos^2\theta} =$

$$= \frac{[1-\cos^2\theta]^2 + \cos^2\theta}{\cos^2\theta} = \frac{1 - 2\cos^2\theta + \cos^4\theta + \cos^2\theta}{\cos^2\theta} =$$

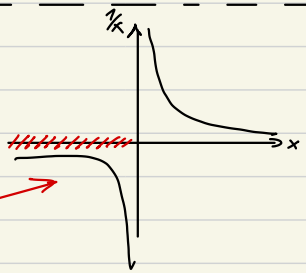
$$= \frac{\cos^4\theta + [1-\cos^2\theta]}{\cos^2\theta} = \frac{\cos^4\theta + \sin^2\theta}{\cos^2\theta} =$$

$$= \cos^2\theta + \tan^2\theta = \tan^2\theta + \cos^2\theta \equiv RHS$$

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a) $\frac{-1}{x} \geq 0 \qquad \frac{1}{x} \leq 0 \Rightarrow \boxed{x < 0}$

$\frac{1}{x} < 0$ when $x < 0$

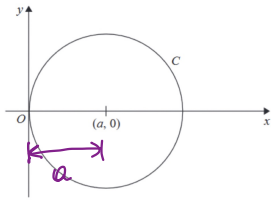



b) $f(x) > 0$

because $\sqrt[2]{}$ can't be $< 0$ but

$f$ can't be $0$ because $\frac{-1}{x} \neq 0$

## QUESTION 13




The diagram shows a circle which touches the $y$-axis and has a centre at the point $(a, 0)$ where $a$ is a positive constant.

Given that the point $(4, -3)$ lies on the circle, find the value of $a$.

Centre: $(a, 0)$ radius $= a$ hence

$a > 0$

$(x-a)^2 + y^2 = a^2;$

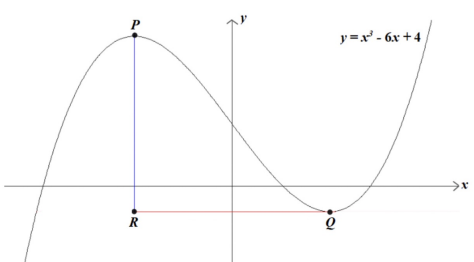
$(4-a)^2 + (-3)^2 = a^2;$

$16 - 8a + a^2 + 9 = a^2;$ $\quad 8a = 25$ $\quad \boxed{a = \frac{25}{8}}$

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(Calculator allowed but this can be done without a calculator)

The graph of $y = x^3 - 6x + 4$ is shown below, where $P$ and $Q$ are stationary points of the curve.




## QUESTION 14

The tangent to the curve at $Q$ and the normal to the curve at $P$ intersect at the point $R$. Find the area of triangle $PRQ$

Stationary points are when $\frac{dy}{dx} = 0$ hence

$\frac{dy}{dx} = 3x^2 - 6$ hence

$3x^2 - 6 = 0;$ $\quad x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{2}$

$$y(-\sqrt{2}) = (-\sqrt{2})^3 - 6(-\sqrt{2}) + 4 = 4 + 4\sqrt{2}$$

$$\Rightarrow P(-\sqrt{2};\ 4+4\sqrt{2})$$

$$y(\sqrt{2}) = (\sqrt{2})^3 - 6(\sqrt{2}) + 4 = 4 - 4\sqrt{2}$$

$$\Rightarrow Q(\sqrt{2};\ 4-4\sqrt{2})$$

$$A_{PRQ} = \frac{1}{2} \times PR \times RQ = \frac{1}{2}\left[4+4\sqrt{2} - (4-4\sqrt{2})\right]\left[\sqrt{2} - (-\sqrt{2})\right]$$

$$= 16$$

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**QUESTION 15**

a) Factorise $2x^2 - 3x - 14$

b) Hence or otherwise solve the equation

$$2(4x^2 + 9x - 2)^2 - 3(4x^2 + 9x - 2) - 14 = 0$$

a) $2x^2 - 3x - 14 = (2x - 7)(x + 2)$

b) $2(4x^2 + 9x - 2)^2 - 3(4x^2 + 9x - 2) - 14 = 0$

⊙ Let $q = 4x^2 + 9x - 2$

$$2q^2 - 3q - 14 = 0 \Rightarrow (2q - 7)(q + 2) = 0$$

$$\Rightarrow \begin{cases} q = \frac{7}{2} & (I) \\ q = -2 & (II) \end{cases}$$

(I) $4x^2 + 9x - 2 = \frac{7}{2} \quad | \times 2$

$8x^2 + 18x - 4 - 7 = 0; \qquad 8x^2 + 18x - 11 = 0$

$(4x + 11)(2x - 1) = 0$

$$\Rightarrow \begin{cases} x = \frac{-11}{4} \\ x = \frac{1}{2} \end{cases}$$

(II) $4x^2 + 9x - 2 = -2$

$4x^2 + 9x = 0; \qquad x(4x + 9) = 0$

$$\Rightarrow \begin{cases} x = 0 \\ x = \frac{-9}{4} \end{cases}$$

$675x^4 - 2505x^3 + 2407x^2 - 63x - 162 \equiv (ax - b)(cx + d)(ex - c)^2$

$a, b, c, d$ and $e$ are different positive integers. Find their values.

## QUESTION 16

Calculator says that the roots are

$\frac{9}{5}, \frac{1}{3}, \frac{-2}{9}$

hence the factorised form will look like this:

$675(\ldots x - 1)(\ldots x + 2)(\ldots x - 9)(\ldots x + ?)$

repeated root

but all of the numbers when multiplied together must give us $-162$ hence

$-1 \times +2 \times -9 \times ? = -162$;

$? = \frac{-162}{-1 \times 2 \times -9} = -9 \Rightarrow$ hence $\frac{9}{5}$ is the repeated root

hence

$$675\left(x-\frac{1}{3}\right)\left(x+\frac{2}{9}\right)\left(x-\frac{9}{5}\right)\left(x-\frac{9}{5}\right)$$

$$= 3\times3\times3\times5\times5\left(x-\frac{1}{3}\right)\left(x+\frac{2}{9}\right)\left(x-\frac{9}{5}\right)\left(x-\frac{9}{5}\right)$$

$$= (3x-1)(9x+2)(5x-9)(5x-9)$$

$$= (3x-1)(9x+2)(5x-9)^2$$

$$\Rightarrow \begin{cases} a=3 \\ b=1 \\ c=9 \\ d=2 \\ e=5 \end{cases}$$